Вычислительные технологии, 2023, том 28, № 2, с. 72–88. © ФИЦ ИВТ, 2023
Computational Technologies, 2023, vol. 28, no. 2, pp. 72–88. © FRC ICT, 2023

ISSN 1560-7534
eISSN 2313-691X

COMPUTATIONAL TECHNOLOGIES

DOI:10.25743/ICT.2023.28.2.007

Optimizing a linear fractional function with interval coefficients over an integer efficient set under chance constraints

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Received June 09, 2022, revised October 31, 2022, accepted November 04, 2022.

In this paper, we present an exact algorithm for optimizing a linear fractional function with interval coefficients over the integer efficient set of a chance constrained multiple objective stochastic integer linear programming (CCMOSILP) problem. At first, a convex combination of the left and right values of the interval coefficients are used in place of the intervals and consequently the problem is reduced to a linear deterministic programming problem. Then we convert the CCMOSILP problem into a deterministic problem by using known distribution function of random variables. The basic idea of the computation phase of the algorithm is to solve the problem using a sequence of progressively more constrained integer linear fractional programs that progressively improves the value of the linear criteria and eliminates undesirable points from further consideration. To demonstrate the proposed algorithm a numerical example is solved.

Keywords: rractional programming, multiobjective stochastic integer, chance constraint programming, interval coefficients, efficient set.

Citation: Younsi-Abbaci L., Moulaï M. Optimizing a linear fractional function with interval coefficients over an integer efficient set under chance constraints. Computational Technologies. 2023; 28(2):72–88. DOI:10.25743/ICT.2023.28.2.007.

Introduction

Multiobjective stochastic optimization is one of the important fields of study in operations research. Many real-world problems involve multiple objectives with random parameters. Due to the conflict between objectives, finding a feasible solution that simultaneously optimizes all objectives is usually impossible. Consequently, in practice, decision makers want to explore and understand the trade off between objectives before choosing a suitable solution.

The multiobjective stochastic program (MOSP) problem was studied by Teghem et al. [1] who presented interactive methods in stochastic programming, PROTRADE of Goicoechea et al. [2], and PROMISE of Urli and Nadeau [3]. These methods have been successfully tested in real world contexts. Ben Abdelaziz et al. [4] proposed a compromise chance constrained approach to solve a MOSP portfolio selection problem. The chance constrained programming (CCP) technique is one which can be used to solve problems involving chance constraints, i. e., constraints having finite probability of being violated. The CCP was originally developed by Charnes and Cooper [5] and has, in recent years, been generalized in several directions and has various applications.

We consider the chance constraints multiple objective stochastic integer linear programming (CCMOSILP) problem [6]:

$$(CCMOSILP)\begin{cases} \min & Z^k = \sum\limits_{i=1}^{n} C_i^k(\xi)x_i, \text{ where } k = 1, \dots, K, \\ \text{s. t.} & X = (x_1, \dots, x_n) \in D, \\ & \mathbb{P}(T(\xi)X \geq h(\xi)) \geq \alpha. \end{cases} \tag{1}$$

We assume throughout the paper that $D_{cs} = D \cap [\mathbb{P}(T(\xi)X \geq h(\xi)) \geq \alpha] \neq \emptyset$, where $D = S \cap \mathbb{Z}$ which $S = \{X \in \mathbb{R}^n : AX = b, X \geq 0\}$ is a nonempty bounded polyhedron. S is the set of deterministic constraints with A is $(m \times n)$ matrix and b is m vector; C^k, T and h are random matrices of dimension $(1 \times n)$, $(m \times n)$ and $(m \times 1)$ respective defined on some probability space $(\Xi, \mathcal{A}, \mathbb{P})$, with Ξ is a set of outcomes ξ (e. g. the results of random experiments), a collection $\mathcal{A}$ of subsects $\mathbf{A} \subset \Xi$ called events and $\mathbb{P}$ is the partially known probability distribution that assigns to each $\mathbf{A} \in \mathcal{A}$ the probability of occurrence, α are specified probabilities. Let $\mathbf{E}_{cs}$ denote the set of efficient solutions, whose definition will be given in the next section.

In many situations, the decision maker faces a large number of different efficient solutions and the selection of his/her preferred solutions becomes a very hard task. A way of assessing some preferred solution is by optimizing a function (utility function written as a function of decision variables), particularly linear, optimization over the efficient set, an appropriate approach that has received increasing attention in recent years. In [7] Philip first studied the problem and suggested an algorithm based on moving to adjacent efficient vertices when the function is a linear function. Later, Isermann and Steuer [8] outlined a similar procedure for solving the problem of optimizing over the efficient set, where the objective function is one of the multiobjective linear programming. Abbas and Chaabane (2006) [9], proposed a method for the optimization over the efficient set of a multiple objective integer linear programming (MOILP), where different types of cuts are imposed in such a way that the improvement of the objective value at each iteration is guaranteed. Jorge [10], Chaabane and Pirlo [11], developed another approach that defines a sequence of progressively more constrained single-objective integer problems that successively eliminates undesirable points. Zerdani and Moulaï [12] developed an approach that optimizes an arbitrary linear function over an integer efficient set of multi objective linear fractional programming (MOLFP) problem without explicitly having to enumerate all the efficient solutions. Recently, Younsi and Moulaï [13] have optimized a stochastic linear over the efficient set of the multi objective stochastic integer linear programming problem, it is based on Jorge's approach [10] with the concepts L-shaped integer method, using an augmented weighted Tchebychev program to generate the set of nondominated objective vectors.

In management science, there are numerous decision marking problems where the objective functions are linear fractional functions with interval coefficients. This type of functions can be found in game theory, portfolio selection, agriculture based management systems, in which the coefficients are not certain when they are modelled mathematically. The basic problem P_E that we investigate is to minimize a main linear fractional function with interval coefficients Φ over the set $\mathbf{E}_{cs}$:

$$(P_E)\begin{cases} \min \quad \Phi(X) = \dfrac{\delta + \sum\limits_{i=1}^{n} P_i x_i}{\beta + \sum\limits_{i=1}^{n} Q_i x_i}, \\ \text{s.t.} \quad X \in \mathbf{E}_{cs}. \end{cases} \tag{2}$$

Here δ, P_i, β and Q_i are an intervals which represents the uncertain coefficients of the objective function, $\forall i = 1, \ldots, n$, with $\delta = [\delta^1, \delta^2]$, $\beta = [\beta^1, \beta^2]$, $P_i = [p_i^1, p_i^2]$ and $Q_i = [q_i^1, q_i^2]$.

The main difficulty of the problem arises from the nonconvexity of the efficient set $\mathbf{E}_{cs}$, which is the union of several faces of D_{cs} (the problem is to be solve without solving CCMOSILP).

Associated with P_E, the relaxed problem is

$$(R_{cs})\begin{cases} \min \quad \Phi(X) = \dfrac{\delta + \sum\limits_{i=1}^{n} P_i x_i}{\beta + \sum\limits_{i=1}^{n} Q_i x_i}, \\ \text{s.t.} \quad X \in D_{cs}. \end{cases} \tag{3}$$

It has also been assumed that $\beta + \sum\limits_{i=1}^{n} Q_i x_i > 0$ for all $X = (x_1, \ldots, x_n) \in D_{cs}$.

In this paper, we focus on the problem of optimizing a linear fractional function with interval coefficients Φ, over the efficient set of a MOSILP with a joint chance constraint. We address the general case where Φ is reduced into a deterministic linear fractional function. Then, the stochastic objective function is converted into deterministic function. We also transform the chance constraint into a deterministic constraints by using known inverse distribution function. A direct approach could consist of finding all efficient solutions of the CCMOSILP problem and then finding the best value of Φ on that set. This approach is not appropriate for practical purposes, because of the difficulty of determining the set of all efficient solutions. We thus propose an implicit technique that avoids searching for all efficient solutions but guarantees finding one that minimize Φ.

The structure of the paper is organized as follows: Section 1 presents the formulation of chance constrained and describes the process of transforming CCMOSILP problem into an equivalent deterministic and compiles the basic result used throughout the manuscript. Section 2 presents new approach for reducing the fractional function with interval coefficients into an deterministic function. A proposition is provided to support this approach. Section 3 is devoted to proposed the different steps of present method and the algorithm. Two propositions are provided to justify finiteness and convergence of the algorithm. An extensive numerical example is solved in Section 4 to show the optimum of proposed problem (2).

1. Chance constrained and efficiency testing

1.1. Chance constrained programming

Use of chance constrained programming (CCP) introduces a new requirement upon decision makers. This approach was first introduced by Charnes et al. [14], where the objective is often an expectational functional as we used earlier (the E-model), or the V-model minimizes the generalized mean square of the objective functions, or the P-model maximizes the probability of aspiration levels of the objective functions. Another variation includes an objective that is a quantile of a random function [15]. Ben Abdelaziz et al. [4] proposed a compromise chance constrained approach to solve multiobjective stochastic programming problem of a portfolio selection.

Since $C_i^k(\xi)$ are uniformly distributed random variables, the k^{th} objective function, $Z^k(X)$, will also be uniformly distributed random variable. The mean of Z^k is given by

$$\widetilde{Z}^k = Esp(Z^k) = \sum_{i=1}^{n} Esp[C_i^k(\xi)]x_i, \quad k = 1, \ldots, K,$$

where $Esp(C_i^k(\xi))$ is the mean value of $C_i^k(\xi)$, $k = 1, \ldots, K$, denoted $Esp(C_i^k(\xi)) = \widetilde{C}_i^k$, assuming that all coefficients of $\widetilde{C}_i^k$ are integers.

Thus, a deterministic linear program with a single chance constraint problem can be formulated as follows:

$$(MOSILP)\begin{cases} \min & \widetilde{Z}^k = \sum\limits_{i=1}^{n} \widetilde{C}_i^k x_i, \quad k = 1, \ldots, K, \\ \text{s. t.} & X \in D, \\ & \mathbb{P}(T_j(\xi)X \geq h_j(\xi)) \geq \alpha_j, \quad j = 1, \ldots, J, \end{cases} \tag{4}$$

where α_j is some confidence level, typically 90 or 95 %, at least, the satisfaction degree on the realization of the uncertain constraints.

For stochastic linear programs with single chance constraint, convexity statements have been derived without the joint convexity assumption on $h_j(\xi) - T_j(\xi)X$; for special distributions and special intervals for the values of α_j. In particular, when each j corresponds to a distinct linear constraint and T_j is a fixed row vector, then obtaining a deterministic equivalent of (4) is fairly straightforward. In this case, $\mathbb{P}(T_jX \geq h_j(\xi)) = F_j(T_jX)$, where $F_j(\cdot)$ and $F_j^{-1}(\cdot)$ represent the distribution function and the inverse distribution function of a uniform variable h_j respectively, we have $\mathbb{P}(T_jX \geq h_j(\xi)) = F_j(T_jX) \geq \alpha_j$ or equivalently $T_jX \geq F_j^{-1}(\alpha_j)$, where $F_j^{-1}(\alpha_j)$ is assumed to be the smallest real value η such that $F_j(\eta) \geq \alpha_j$. Hence in this special case any single chance constraint turns out to be just a linear constraint, and the only additional work to do is to compute $F_j^{-1}(\alpha_j)$.

Thus, the chance constrained programming problem can be stated as a deterministic linear programming problem:

$$(MCP)\begin{cases} \min & \widetilde{Z}^k = \sum\limits_{i=1}^{n} \widetilde{C}_i^k x_i, \quad k = 1, \ldots, K, \\ \text{s. t.} & X \in D, \\ & T_jX \geq F_j^{-1}(\alpha_j), \quad j = 1, \ldots, J. \end{cases} \tag{5}$$

We assume throughout the paper that

$$D_{cs} = \{X \in \mathbb{R}^n : AX = b, T_jX \geq F_j^{-1}(\alpha_j), j = 1, \ldots, J, X \geq 0, \text{integer}\}$$

is not empty.

In the sense of MCP programming, K objective are usually simultaneously each other in nature and concept of optimal solution gives place to concept of Pareto optimal (efficient, non dominated), for which the improvement of one objective function is attained only by sacrificing another objective function. The solution to the problem (5) is to find all solutions that are efficient in the sense of the following definition:

Definition 1.1. A point $X^* \in D_{cs}$ is said to be an efficient solution for (5) if and only if there does not exist another point $X^{(1)} \in D_{cs}$ such that $\widetilde{Z}^k(X^{(1)}) \leq \widetilde{Z}^k(X^*)$, $k \in \{1, \ldots, K\}$ and $\widetilde{Z}^k(X^{(1)}) < \widetilde{Z}^k(X^*)$ for at least one $k \in \{1, \ldots, K\}$.

In this subsection we pay attention to some basic results which can help the reader to understand the algorithm in Section 3.

1.2. Efficiency testing

The following result [16] is used in various steps of the algorithm to test the efficiency of a given feasible solution of problem (5).

Let X^0 be an arbitrary element of the region D_{cs}; $X^0 \in \mathbf{E}_{cs}$ if and only if the optimal value of the objective function Θ is null in the following integer linear programming problem:

$$(P(X^0)) \begin{cases} \min \quad \Theta = -\sum_{k=1}^{K} \Psi_k, \\ \text{s.t.} \quad \begin{cases} X \in D_{cs}, \\ \widetilde{C}^k X + \Psi_k = \widetilde{C}^k X^0, \\ \Psi^k \geq 0, \quad \text{integer}. \end{cases} \end{cases} \tag{6}$$

As is well-known, if the optimal value $\Theta = 0$, then $X^0 \in \mathbf{E}_{cs}$. Otherwise, any optimal solution $\hat{X}^0$ of (6) is proved to be an efficient solution of (5) and its criterion vector dominates $\widetilde{C}X^0$.

2. Construction of deterministic fractional function

2.1. The basic interval arithmetic

All lower case letters denote real numbers and the upper case letters denote the interval numbers or the closed intervals on $\mathbb{R}$:

$$A = [a^1, a^2] = \{a : a^1 \leq a \leq a^2\},$$

where a^1 and a^2 are the left and right values of the interval A on the real line $\mathbb{R}$, respectively. If $a^1 = a^2$, then $A = [a, a]$ is a real number.

Let $* \in \{+, -, \cdot, \div\}$ be a binary operation on the set of real numbers. If A and B are closed intervals, then

$$A * B = \{a * b : a \in A, b \in B\}$$

defines a binary operation on the set of closed intervals. In the case of division, it is assumed that $0 \notin B$. See [17, 18] for more information about interval arithmetic.

If μ is a scalar, then

$$\mu \cdot A = \mu \cdot [a^1, a^2] = \begin{cases} \mu \cdot [a^1, a^2] & \text{for } \mu \geq 0, \\ \mu \cdot [a^1, a^2] & \text{for } \mu < 0. \end{cases}$$

The extended addition $(+)$ and extended subtraction $(-)$ are defined as follows:

$$A + B = [a^1 + b^1, a^2 + b^2], \quad A - B = [a^1 - b^2, a^2 - b^1].$$

2.2. The best objective

The importance and motivation for converting the function into a deterministic function of problem (3) has been discussed extensively in the literature. For example, Effati and Pakdaman [19] discussed solving procedure of interval valued linear fractional programming model (LFPM). In [20] a generalized confidence interval estimation method is used, to obtain the left and right values of interval estimated linear fractional programming model (INLFPM). Borza et al. [21] proposed a method on variable transformation by Charnes and Cooper and convex combination of the left and right values of the intervals. In this section we offer an approach that consists in reduceing the function of problem (3) into a deterministic fractional function using convex combination.

Proposition 2.1. *In the objective function of problem* (3) *only the left values of the numerator intervals and only the right values of the denominator intervals are used to (obtain) achieve the best objective.*

Proof. Minimize objective function

$$\Phi(X) = \frac{[\delta^1, \delta^2] + \sum_{i=1}^{n} [p_i^1, p_i^2] x_i}{[\beta^1, \beta^2] + \sum_{i=1}^{n} [q_i^1, q_i^2] x_i}$$

is equivalent to minimize its numerator and maximize its denominator. Using the convex combination of the intervals yields the following objective function:

$$\frac{[\delta^1, \delta^2] + \sum_{i=1}^{n} [p_i^1, p_i^2] x_i}{[\beta^1, \beta^2] + \sum_{i=1}^{n} [q_i^1, q_i^2] x_i} = \frac{\lambda_0 \delta^1 + (1-\lambda_0)\delta^2 + \sum_{i=1}^{n} (\lambda_i p_i^1 + (1-\lambda_i) p_i^2) x_i}{(1-\lambda_0)\beta^1 + \lambda_0 \beta^2 + \sum_{i=1}^{n} ((1-\lambda_i) q_i^1 + \lambda_i q_i^2) x_i}.$$

Let $\widehat{X} = (\widehat{x}_1, \ldots, \widehat{x}_n)$ be a point of the feasible region of problem (3), with $\lambda_0, \lambda_i \in [0, 1]$, $(\delta^1 - \delta^2) < 0$, $(p_i^1 - p_i^2) < 0$, $(\beta^2 - \beta^1) > 0$ and $(q_i^2 - q_i^1) > 0$ then the objective function can be written as:

$$\frac{\lambda_0(\delta^1 - \delta^2) + \sum_{i=1}^{n} \lambda_i (p_i^1 - p_i^2) \widehat{x}_i + \left(\delta^2 + \sum_{i=1}^{n} p_i^2 \widehat{x}_i\right)}{\lambda_0(\beta^2 - \beta^1) + \sum_{i=1}^{n} \lambda_i (q_i^2 - q_i^1) \widehat{x}_i + \left(\beta^1 + \sum_{i=1}^{n} q_i^1 \widehat{x}_i\right)}.$$

On the other hand, for all index $i = 1, \ldots, n$:

$$\lambda_0(\delta^1 - \delta^2) + \sum_{i=1}^{n} \lambda_i (p_i^1 - p_i^2) \widehat{x}_i + \left(\delta^2 + \sum_{i=1}^{n} p_i^2 \widehat{x}_i\right) >$$

$$> (\delta^1 - \delta^2) + \sum_{i=1}^{n} (p_i^1 - p_i^2) \widehat{x}_i + \left(\delta^2 + \sum_{i=1}^{n} p_i^2 \widehat{x}_i\right) = \delta^1 + \sum_{i=1}^{n} p_i^1 \widehat{x}_i,$$

and

$$\lambda_0(\beta^2 - \beta^1) + \sum_{i=1}^{n} \lambda_i (q_i^2 - q_i^1) \widehat{x}_i + \left(\beta^1 + \sum_{i=1}^{I} q_i^1 \widehat{x}_i\right) <$$

$$< (\beta^2 - \beta^1) + \sum_{i=1}^{I} (q_i^2 - q_i^1) \widehat{x}_i + \left(\beta^1 + \sum_{i=1}^{n} q_i^1 \widehat{x}_i\right) = \beta^2 + \sum_{i=1}^{n} q_i^2 \widehat{x}_i.$$

The following inequality is obtained

$$\frac{\lambda_0(\delta^1-\delta^2)+\sum\limits_{i=1}^{n}\lambda_i(p_i^1-p_i^2)\widehat{x}_i+\left(\delta^2+\sum\limits_{i=1}^{n}p_i^2\widehat{x}_i\right)}{\lambda_0(\beta^2-\beta^1)+\sum\limits_{i=1}^{n}\lambda_i(q_i^2-q_i^1)\widehat{x}_i+\left(\beta^1+\sum\limits_{i=1}^{n}q_i^1\widehat{x}_i\right)}>\frac{\delta^1+\sum\limits_{i=1}^{n}p_i^1\widehat{x}_i}{\beta^2+\sum\limits_{i=1}^{n}q_i^2\widehat{x}_i}.$$

The right hand side of the above inequality can be considered as a lower bound for the objective function of the problem (3). Therefore the problem (3) can be reduced to the following problem:

$$(RE_{cs})\begin{cases}\min\quad \Phi(X)=\dfrac{\delta^1+\sum\limits_{i=1}^{n}p_i^1x_i}{\beta^2+\sum\limits_{i=1}^{n}q_i^2x_i},\\ \text{s.t.}\quad X\in D_{cs}\end{cases}\tag{7}$$

and we write the problem (2) using this proposition, we have the following problem:

$$(P_{E_{cs}})\begin{cases}\min\quad \Phi(X)=\dfrac{\delta^1+\sum\limits_{i=1}^{n}p_i^1x_i}{\beta^2+\sum\limits_{i=1}^{n}q_i^2x_i},\\ \text{s.t.}\quad X\in \mathbf{E}_{cs}.\end{cases}\tag{8}$$

□

Different approaches have been proposed in the literature to solve problem (7) integer linear fractional programming problems [22–25]. The approach adopted to solve the problem (7) at the l^{th} iteration, is the Granot's method (see [24]), which is mainly based on the evaluation of the reduced gradient vector $\bar{\gamma}_j$ and Gomory cuts, if necessary, we obtain an integer feasible solution. The following theorem allows us to find the optimal solution of (7).

Theorem 2.1 (see [25])**.** *The point X^1 of D_{cs} is an optimal solution of problem* (7) *if and only if the reduced gradient vector $\bar{\gamma}=\bar{\beta^2}\bar{p^1}-\bar{\delta^1}\bar{q^2}$ is such that $\bar{\gamma}_j\geq 0$ for all $j\in N_l$ (N_l is the set of indices of nonbasic variables of X^1), where $\bar{\beta^2}$, $\bar{p^1}$, $\bar{\delta^1}$ and $\bar{q^2}$ are the updated values of β^2, p^1, δ^1 and q^2 respectively.*

3. Description of the method

The considered problem presents three principal basic ideas. First one, we reduce the linear objective function with intervals coefficients by the best objective. The second idea is to compute the inverse of the distribution function of variable $h_j(\xi)$ for transformed the chance constraints of the problem (5). Finally, we characterize an efficient solution by solving the efficiency test (problem (6), see [16]) and we reduce progressively the admissible domain by adding more constraints in order to present the detailed steps of algorithm that solves problem (8).

Step 1. Starting with an optimal solution of an (7) problem and successive Gomory cuts, if necessary, we obtain an integer feasible solutions. The obtained solution is then tested for efficiency by solving $(P(X^1))$, terminating if it finds that it is efficient, or obtaining an efficient solution $\hat{X}^1$, whose criterion vector dominates $\widetilde{C}X^1$.

Step 2. It may happen that the obtained efficient solution is not better than an equivalent efficient solution on the main objective function $\Phi(X)$. Therefore, the following problem has to be solved before reducing the current admissible region.

$$\min\{\Phi(X)|\widetilde{C}X = \widetilde{C}\hat{X}^1, \Phi(X) \leq (\Phi(\hat{X}^1) - e), X \in D_{cs}\},$$

where $e > 0$ is a positive small enough value to avoid falling on the same solution $\hat{X}^1$. If the problem is unfeasible and if $\Phi(\hat{X}^1) \leq \Phi_{opt}$, $X_{opt} = \hat{X}^1$ and $\Phi_{opt} = \Phi(\hat{X}^1)$ have been updated as a consequence of having found a new better efficient solution of (5). Otherwise, a new efficient solution $\ddot{X}^1$ is then generated and if $\phi(\ddot{X}^1)$ is inferior from the optimal value, update $X_{opt} = \ddot{X}^1$ and $\Phi_{opt} = \Phi(\ddot{X}^1)$.

Step 3. After l steps of the process, the feasible set D_{cs} is reduced gradually by eliminating all dominated solutions by $\widetilde{C}\hat{X}^{l-1}$ (see Sylva and Crema [26]) with the cut $\Phi(X) \leq \Phi_{opt}$ that insure that the new optimal solution X^l of the problem (9) improves the optimum value. The resolution of the following problem (9) enables us to perform this elimination assuming that all coefficients of $\widetilde{C}$ are integers.

$$(RE^l_{cs})\begin{cases} \min \quad \Phi(X) = \dfrac{\delta^1 + \sum\limits_{i=1}^{n} p_i^1 x_i}{\beta^2 + \sum\limits_{i=1}^{n} q_i^2 x_i}, \\ \text{s.t.} \quad X \in H_{cs}, \\ \qquad\; \Phi(X) \leq \Phi_{opt}, \end{cases} \tag{9}$$

where $H_{cs} = D_{cs} - \bigcup\limits_{s=1}^{l} D_s$ and $D_s = \left\{X \in \mathbb{Z}^n | \widetilde{C}X \geq \widetilde{C}X^s\right\}$. $\left\{\widetilde{C}X^s\right\}_{s=1}^{l}$ is a subset of nondominated criteria vectors for problem (5), with $\{X^s, s = 1, \ldots, l-1\}$ are solutions of (5) obtained at iterations 1, 2, ..., $l-1$ respectively.

$$H_{cs} = \begin{cases} \widetilde{C}^k X \leq (\widetilde{C}^k X^s - 1)y^{sk} + M^k(1 - y^{sk}), \\ \sum\limits_{k=1}^{K} y^{sk} \geq 1, \\ y^{sk} \in \{0, 1\}, \\ X \in D_{cs}, \\ \text{for all } k = 1, 2, \ldots, K, \quad s = 1, 2, \ldots, l, \end{cases}$$

where M^k is an upper bound for the k^{th} objective function of problem (5). In practice, M_k can be taken, e.g., as the optimal value of the linear problem $\max\{\widetilde{C}^k X | X \in D_{cs}, X \geq 0, \text{integer}\}$. Note that when $y^{sk} = 0$, the constraint is not restrictive and when $y^{sk} = 1$, a strict improvement is forced in the k^{th} objective function evaluated at $\ddot{X}^l$ or $\hat{X}^l$, and

$$\Phi(X) \leq \Phi_{opt} \Leftrightarrow \frac{\delta^1 + \sum\limits_{i=1}^{n} p_i^1 x_i}{\beta^2 + \sum\limits_{i=1}^{n} q_i^2 x_i} \leq \Phi_{opt} \Leftrightarrow \left(\delta^1 + \sum_{i=1}^{n} p_i^1 x_i\right) - \Phi_{opt}\left(\beta^2 + \sum_{i=1}^{n} q_i^2 x_i\right) \leq 0 \Leftrightarrow$$

$$\Leftrightarrow \sum_{i=1}^{n} (p_i^1 - \Phi_{opt} q_i^2) x_i \leq (-\delta^1 + \Phi_{opt}\beta^2).$$

Proposition 3.1. *Let $\hat{X}^l$ be an optimal solution to the problem $P(X^l)$ (where X^l is an optimal solution to the problem* (9)*). If $\Phi(\hat{X}) = \Phi(X^l)$ then $\hat{X}^l$ is an optimal solution to the problem* (8)*.*

Proof. Let us suppose on the contrary that $\hat{X}^l$ is not an optimal solution of (8). There exists another point denoted $\bar{X} \in E_{cs}$ such that $\Phi(\bar{X}) < \Phi(\hat{X}^l)$. But on the other hand $\Phi(\hat{X}^l) = \Phi(X^l)$, therefore $\Phi(\bar{X}) < \Phi(X^l)$. Thus, it is contradicting the assumption that X^l is an optimal solution to the problem (7). □

3.1. Algorithm

Step 0. Initialization:

- In chance constrained programming, we transform the chance constraints into deterministic constraints by using known inverse distribution function. And using the mathematical means we determine the compromise function for each criterion.
- Using the lower bound of the objective function (the best objective) instead of the fractional linear function with interval coefficients.
- Solving the linear problem $\max\{\widetilde{C}^k X | X \in D_{cs}\}$ for determined M_k the optimal value of objective function k.
- $l = 1$, $\Phi_{opt} = +\infty$ and $H^1_{cs} = D_{cs}$, $e = 0.01$.
- Find an optimal solution X^1 of (7).

Step 1.

- If $\Theta = 0$, then X^l is an optimal solution of (8), the algorithms is terminated. Else, find an optimal solution $\hat{X}^l$ of $(P(X^l))$.
- If $\Phi(X^l) = \Phi(\hat{X}^l)$, then $\hat{X}^l$ solves (8), the algorithms is terminated. Otherwise, go to Step 2.

Step 2. Find an equivalent efficient solution improving the main objective, with the same criteria vector by solving the problem

$$\min\left\{\Phi(X) | \widetilde{C}X = \widetilde{C}\hat{X}^l,\ \Phi(X) \leq (\Phi(\hat{X}^l) - 0.01),\ X \in D_{cs}\right\}. \tag{10}$$

- If the problem (10) is unfeasible, if $\Phi(\hat{X}^l) \leq \Phi_{opt}$, set $X_{opt} = \hat{X}^l$ and $\Phi_{opt} = \Phi(\hat{X}^l)$. Go to Step 3.
- Otherwise (problem (10) is feasible), let $\ddot{X}^l$ be an optimal solution of the problem (10), if $\Phi(\ddot{X}^l) \leq \Phi_{opt}$, set $X_{opt} = \ddot{X}^l$ and $\Phi_{opt} = \Phi(\ddot{X}^l)$. Go to Step 3.

Step 3. Set $l = l + 1$, and solve the problem

$$(RE^l_{cs}) \begin{cases} \min \quad \Phi = \dfrac{\delta^1 + \sum\limits_{i=1}^{n} p^1_i x_i}{\beta^2 + \sum\limits_{i=1}^{n} q^2_i x_i}, \\ \text{s.t.} \quad X \in H^l_{cs} = \begin{cases} X \in H^{l-1}_{cs}, \\ \widetilde{C}^k X - (\widetilde{C}^k X_{opt} - 1 - M^k) y^{lk} \leq M^k, \\ \sum\limits_{k=1}^{K} y^{lk} \geq 1, \\ y^{lk} \in \{0, 1\}, \quad k = 1, 2, \dots, K, \end{cases} \\ \quad \sum\limits_{i=1}^{n} (p^1_i - \Phi_{opt} q^2_i) x_i \leq -\delta^1 + \Phi_{opt}\beta^2. \end{cases} \tag{11}$$

- If $H^l_{cs} = \emptyset$. The algorithms is terminated. X_{opt} is an optimal solution for the linear fractional program (8).
- Otherwise, let X^l be an optimal solution of the problem (11). Solve the problem $P(X^l)$ and go to Step 1.

Proposition 3.2. *This algorithm solves problem* (8) *in a finite number of steps.*

Proof. After transforming the chance constrained programming into an equivalent deterministic constraint, the set D_{cs} of feasible solution of problem, being compact contains a finite number of integer solutions. During iteration k, either a cut $\Phi(X) \leq \Phi_{opt}$ is applied, the domain is strictly reduced and one new efficient solution is generated, with three points of stop, leading to convergence of the algorithm in a finite number of steps. □

4. Example illustrative

Let the main problem be:

$$(P_E)\begin{cases} \min \quad \Phi(X) = \dfrac{[1,10]x_1 + [1,4]x_2 + [-1,1]}{[1,5]x_1 + [0.9,1]x_2 + [-1.5,-1]}, \\ \text{s.t.} \quad X = (x_1, x_2) \in \mathbf{E}_{cs}. \end{cases}$$

The problem is reduced to the following problem:

$$(P_{E_{cs}})\begin{cases} \min \quad \Phi(x) = \dfrac{x_1 + x_2 - 1}{5x_1 + x_2 - 1}, \\ \text{s.t.} \quad X \in \mathbf{E}_{cs}. \end{cases}$$

Initialization: we look at the following stochastic multiobjective linear integer program with chance constraints:

$$\begin{cases} \min \quad Z^1 = c^1_1(\xi)x_1 + c^1_2(\xi)x_2, \\ \min \quad Z^2 = c^2_1(\xi)x_1 + c^2_2(\xi)x_2, \\ \text{s.t.} \quad x_1 \leq 5, \\ \qquad x_1 + x_2 \leq 10, \\ \qquad -x_2 \geq h_1(\xi), \\ \qquad 3x_1 + 2x_2 \geq h_2(\xi), \\ \qquad X > 0, \ \text{integer}. \end{cases}$$

Moreover, the density functions for the random coefficients $c^k_i(\xi)$ are uniformly distributed as follows:

$$c^1_1(\xi) \rightsquigarrow U[-6,2], \quad c^1_2(\xi) \rightsquigarrow U[-8,10], \quad c^2_1(\xi) \rightsquigarrow U[-2,4], \quad c^2_2(\xi) \rightsquigarrow U[-3,-1].$$

Compute the mean value of random coefficients for both objectives functions:

$$\widetilde{Z}^1 = Esp(Z^1(x,\xi)) = Esp(C^1_1(\xi))x_1 + Esp(C^1_2(\xi))x_2 = \frac{1}{2}(2-6)x_1 + \frac{1}{2}(10-8)x_2 = -2x_1 + x_2,$$

$$\widetilde{Z}^2 = Esp(Z^2(x,\xi)) = Esp(C^2_1(\xi))x_1 + Esp(C^2_2)(\xi)x_2 = \frac{1}{2}(-2+4)x_1 + \frac{1}{2}(-3-1)x_2 = x_1 - 2x_2.$$

The density functions of both random variables $h_1(\xi)$ and $h_2(\xi)$ are uniform as follows.

$$h_1(\xi) \rightsquigarrow U[-10,-7] \quad \text{and} \quad h_2(\xi) \rightsquigarrow U[1,6].$$

We assume that the desirable safety probability 95 % is realized.

$$\begin{gathered}\mathbb{P}(-x_2 \geq h_1(\xi)) \geq 0.95 \Longrightarrow F_1(-x_2) \geq 0.95 \Longrightarrow -x_2 \geq F_1^{-1}(0.95),\\ \mathbb{P}(3x_1 + 2x_2 \geq h_2(\xi)) \geq 0.95 \Longrightarrow F_2(3x_1 + 2x_2) \geq 0.95 \Longrightarrow 3x_1 + 2x_2 \geq F_2^{-1}(0.95),\\ F_1^{-1}(\alpha) = a + \alpha(b-a) \Rightarrow F_1^{-1}(0.95) = -10 + 0.95(-7+10) = -7.15,\\ F_2^{-1}(\alpha) = a + \alpha(b-a) \Rightarrow F_2^{-1}(0.95) = 1 + 0.95(6-1) = 5.75.\end{gathered}$$

Therefore, the chance constraints of the CCP problem (12) are equivalent to the following constraints:

$$\begin{cases} -x_2 \geq -7.15,\\ 3x_1 + 2x_2 \geq 5.75.\end{cases}$$

We get the following deterministic equivalent problem:

$$(MCP)\begin{cases} \min & \widetilde{Z}^1 = -2x_1 + x_2,\\ \min & \widetilde{Z}^2 = x_1 - 2x_2,\\ \text{s.t.} & x_1 \leq 5,\\ & x_1 + x_2 \leq 10,\\ & -x_2 \geq -7.15,\\ & 3x_1 + 2x_2 \geq 5.75,\\ & X > 0,\ \text{integer}.\end{cases} \tag{12}$$

The relaxed problem is

$$(RE_{cs}^1)\begin{cases} \min & \Phi(x) = \dfrac{x_1 + x_2 - 1}{5x_1 + x_2 - 1},\\ \text{s.t.} & X \in D_{cs},\end{cases} \tag{13}$$

where $D_{cs} = \{x_1 \leq 5, x_1 + x_2 \leq 10, -x_2 \geq -7.15, 3x_1 + 2x_2 \geq 5.75, X > 0, \text{integer}\}$. We take $\Phi_{opt} = +\infty$, $H_{cs}^1 = D_{cs}$, $e = 0.01$ and $l = 1$. Alter solving $\max\{\widetilde{C}^k X | X \in D_{cs}\}$, $(k = 1, 2)$, we set $(M^1, M^2) = (7, 5)$.

The concept of optimal solution can be characterized in many ways. For a geometric analysis of the fractional programs, $r(0, 1)$ is the rotation point, with the arrow circular denote the gradient of linear fractional function (see Fig. 1).

Iteration 1

Step 1. The relaxed problem (13) is solved. The optimal solution is $\Phi^1 = 1/9$ for $X^1 = (2, 0)$. In order to test the efficiency of X^1, we solve problem (14) which is the vector criterion corresponding $\widetilde{Z}(X^1) = (-4, 2)$:

$$(P(X^1))\begin{cases} \min & \Theta = -\Psi_1 - \Psi_2,\\ & X \in D_{cs},\\ & -2x_1 + x_2 + \Psi_1 = -4,\\ & x_1 - 2x_2 + \Psi_2 = 2,\\ & \Psi_i \geq 0, i = 1, 2.\end{cases} \tag{14}$$

The optimal value of (14) is $\Theta = -8$, which is achieved at the point $\hat{X}^1 = (5, 5)$. Thus $\hat{X}^1 \in \mathbf{E}_{cs}$ and $X^1 \notin \mathbf{E}_{cs}$.

Step 2. (T^1_{cs}) is defined as:

$$\min\left\{\Phi(x)=\frac{x_1+x_2-1}{5x_1+x_2-1}, \Phi(X)\le\Phi(\hat{X}^1)-0.01, X\in D_{cs}, -2x_1+x_2=-4, x_1-2x_2=2\right\}.$$

The problem (T^1_{cs}) is unfeasible. $\Phi(\hat{X}^1) = 9/29 < \Phi_{opt} = +\infty$, set $X_{opt} = \hat{X}^1$ and $\Phi_{opt} = 9/29$. Go to Step 3.

Step 3. $l := l+1 = 2$. The optimal solution of

$$(RE^2_{cs})\begin{cases} \min \quad \Phi(X) = \dfrac{x_1+x_2-1}{5x_1+x_2-1}, \\ \text{s.t.} \quad H^2_{cs} = \begin{cases} X \in H^1_{cs}, \\ -2x_1 + x_2 + 13y^{1,2} \le 7, & (1) \\ x_1 - 2x_2 + 11y^{2,2} \le 5, & (2) \\ y^{1,2} + y^{2,2} \ge 1, \quad y^{1,2}, y^{2,2} \in \{0,1\}, \end{cases} \\ \qquad -16x_1 + 20x_2 \le 20 \end{cases} \tag{15}$$

is $X^2 = (3,0)$, $Y^2 = (1,0)$, with $\widetilde{Z}^2 = \widetilde{C}X^2 = (-6,3)$ and $\Phi(X^2) = 1/7$ (see Fig. 2). Solve the following problem:

$$(P(X^2))\begin{cases} \min \quad \Theta = -\Psi_1 - \Psi_2, \\ \qquad X \in D_{cs}, \\ \qquad -2x_1 + x_2 + \Psi_1 = -6, \\ \qquad x_1 - 2x_2 + \Psi_2 = 3, \\ \qquad \Psi_i \ge 0, i = 1,2. \end{cases} \tag{16}$$

Iteration 2

Step 1. The optimal value of (16) is $\Theta = -6$, which is achieved at the point $\hat{X}^2 = (5,4)$. Thus $\hat{X}^2 \in \mathbf{E}_{cs}$ and $X^2 \notin \mathbf{E}_{cs}$. $\Phi(X^2) = 1/7 \ne \Phi(\hat{X}^2) = 2/7$, go to Step 2.

Step 2. (T^2_{cs}) is defined as:

$$\min\left\{\Phi(x)=\frac{x_1+x_2-1}{5x_1+x_2-1}, \Phi(X)\le\Phi(\hat{X}^2)-0.01, x\in D_{cs}, -2x_1+x_2=-6, x_1-2x_2=-3\right\}.$$

The problem (T^2_{cs}) is unfeasible. $\Phi(\hat{X}^2) = 2/7 < \Phi_{opt} = 9/29$, set $X_{opt} = \hat{X}^2$ and $\Phi_{opt} = 2/7$. Go to Step 3.

Step 3. $l := l+1 = 3$. The optimal solution of

$$(RE^3_{cs})\begin{cases} \min \quad \Phi(X) = \dfrac{x_1+x_2-1}{5x_1+x_2-1}, \\ \text{s.t.} \quad H^3_{cs} = \begin{cases} X \in H^2_{cs}, \\ -2x_1 + x_2 + 14y^{1,3} \le 7, & (3) \\ x_1 - 2x_2 + 11y^{2,3} \le 5, & (4) \\ y^{1,3} + y^{2,3} \ge 1, \quad y^{1,3}, y^{2,3} \in \{0,1\}, \end{cases} \\ \qquad -3x_1 + 5x_2 \le 5 \end{cases}$$

is $X^3 = (4,0)$, $Y^3 = (1,0)$, with $\widetilde{Z}^3 = \widetilde{C}X^3 = (-8,4)$ and $\Phi(X^3) = 3/19$ (see Fig. 3). Solve the following problem:

$$(F(X^3))\begin{cases} \min \quad \Theta = -\Psi_1 - \Psi_2, \\ \qquad X \in D_{cs}, \\ \qquad -2x_1 + x_2 + \Psi_1 = -8, \\ \qquad x_1 - 2x_2 + \Psi_2 = 4, \\ \qquad \Psi_i \ge 0, \quad i = 1,2. \end{cases}$$

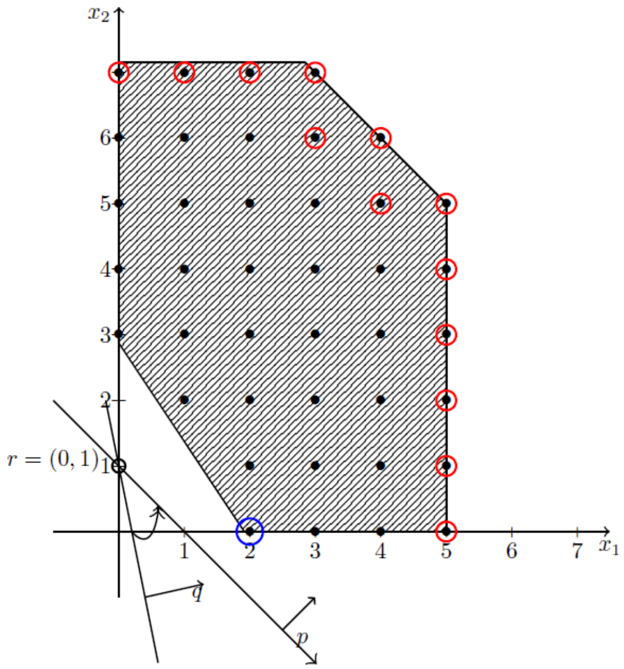

Fig. 1. Admissibility domain without stochastic constraint H^1_{cs}

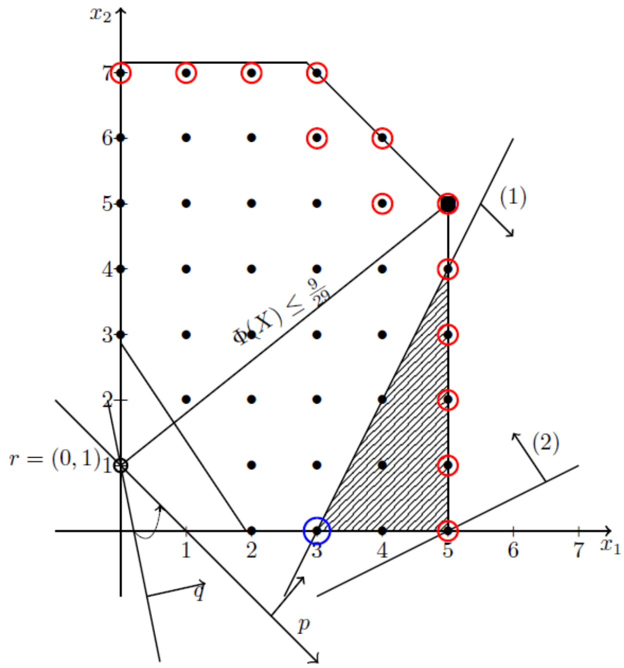

Fig. 2. Admissibility domain without stochastic constraint H^2_{cs}

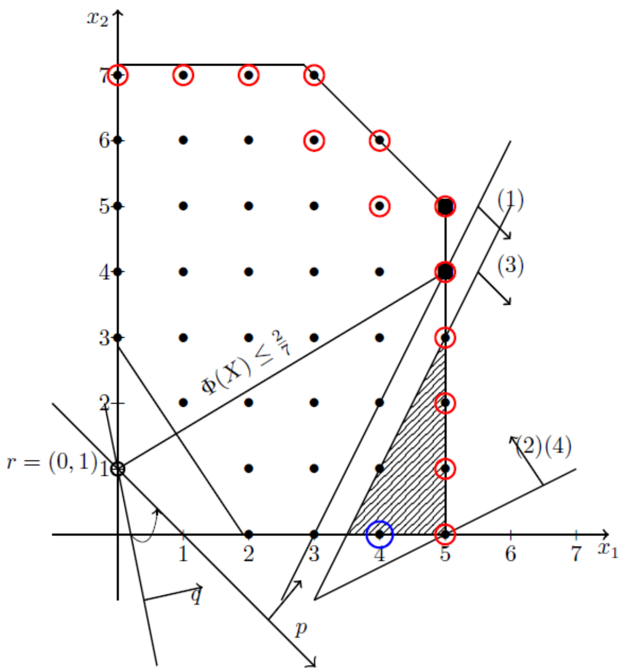

Fig. 3. Admissibility domain without stochastic constraint H^3_{cs}

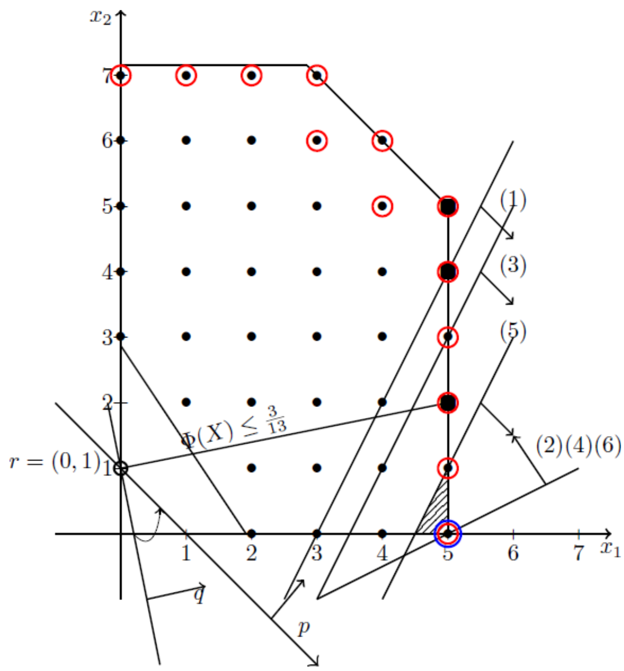

Fig. 4. Admissibility domain without stochastic constraint H^4_{cs}

Iteration 3

Step 1. The optimal value of $(P(X^3))$ is $\Theta = -3$, which is achieved at the point $\hat{X}^3 = (5, 2)$. Thus $\hat{X}^3 \in \mathbf{E}_{cs}$ and $X^3 \notin \mathbf{E}_{cs}$, $\Phi(X^3) = 3/19 \neq \Phi(\hat{X}^3) = 3/13$. Go to Step 2.

Step 2. (T^3_{cs}) is defined as:

$$\min\left\{\Phi(x)=\frac{x_1+x_2-1}{5x_1+x_2-1}, \Phi(X)\leq\Phi(\hat{X}^3)-0.01, X\in D_{cs}, -2x_1+x_2=-8, x_1-2x_2=1\right\}.$$

The problem (T^3_{cs}) is unfeasible. $\Phi(\hat{X}^3) = 3/13 < \Phi_{opt} = 2/7$, set $X_{opt} = \hat{X}^3$ and $\Phi_{opt} = 3/13$. Go to Step 3.

Step 3. $l := l + 1 = 4$. The optimal solution of

$$(RE^4_{cs})\begin{cases} \min \quad \Phi(X) = \dfrac{x_1 + x_2 - 1}{5x_1 + x_2 - 1}, \\ \text{s.t.} \quad H^4_{cs} = \begin{cases} X \in H^3_{cs}, \\ -2x_1 + x_2 + 16y^{1,4} \leq 7, & (5) \\ x_1 - 2x_2 + 5y^{2,4} \leq 5, & (6) \\ y^{1,4} + y^{2,4} \geq 1, \quad y^{1,4}, y^{2,4} \in \{0, 1\}, \end{cases} \\ \qquad -3x_1 + 10x_2 \leq 10 \end{cases}$$

is $X^4 = (5, 0)$, $Y^4 = (1, 0)$, with $\widetilde{Z}^4 = \widetilde{C}X^4 = (-10, 5)$ and $\Phi(X^4) = 1/6$ (see Fig. 4). Solve the following problem:

$$(P(X^4))\begin{cases} \min \quad \Theta = -\Psi_1 - \Psi_2, \\ \qquad X \in D_{cs}, \\ \qquad -2x_1 + x_2 + \Psi_1 = -10, \\ \qquad x_1 - 2x_2 + \Psi_2 = 5, \\ \qquad \Psi_i \geq 0, \quad i = 1, 2. \end{cases} \tag{17}$$

Iteration 4

Step 1. The optimal value of (17) is $\Theta = 0$. Therefore, $\hat{X}^4 = (5, 0) \in \mathbf{E}_{cs}$. This makes the algorithm stop, leaving us with $X_{opt} = \hat{X}^4$ as an optimal solution of (8), as expected.

The set of all efficient solutions of the problem (5) is

$$\mathbf{E}_{cs} = \{(0, 7), (1, 7), (2, 7), (3, 7), (3, 6), (4, 6), (4, 5), (5, 5), (5, 4), (5, 3), (5, 2), (5, 1), (5, 0)\}.$$

Whereas, the proposed algorithm optimizes the linear fractional function $\phi(X)$ without having to pass by all these solutions but only by $\{(5, 5), (5, 4), (5, 2), (5, 0)\}$.

Conclusion

The uncertainty in real-world decision making originates from several sources. In this work, we have made our contribution in stochastic fractional optimizing over the efficient set of CCMOSILP. This problem has not been yet studied in the literature. Initially, the minimization problem (2) with uncertain coefficients of the objective function was reduced to a deterministic problem using the linear combination. We have constructed an equivalent deterministic model corresponding to the CCMOSILP problem. The proposed algorithm solves the deterministic version of the problem (2) by using a sequence of progressively more constrained and the cut of $\phi(X) \leq \phi_{opt}$ without having to enumerate all the efficient solutions. A number of propositions are provided to support finiteness and convergence properties.

For further research, we can consider the lower and upper values of interval estimated linear fractional programming model obtained by using generalized confidence interval estimation method. In real decision problems, in particular, it will be interesting to use the mutiobjective stochastic transportation problems with interval coefficients which have the same formulation as problem (1). In our opinion, a new study based on the development of rational experiment.

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Вычислительные технологии, 2023, том 28, № 2, с. 72–88. © ФИЦ ИВТ, 2023 ISSN 1560-7534
Computational Technologies, 2023, vol. 28, no. 2, pp. 72–88. © FRC ICT, 2023 eISSN 2313-691X

ВЫЧИСЛИТЕЛЬНЫЕ ТЕХНОЛОГИИ

DOI:10.25743/ICT.2023.28.2.007

Оптимизация дробно-линейной функции с интервальными коэффициентами по целочисленному эффективному набору при вероятностных ограничениях

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Поступила 09 июня 2022 г., доработана 31 октября 2022 г., принята в печать 04 ноября 2022 г.

Аннотация

В этой статье мы представляем точный алгоритм оптимизации дробно-линейной функции с интервальными коэффициентами по целочисленному эффективному множеству задачи стохастического целочисленного линейного программирования с множественными целями и вероятностными ограничениями (CCMOSILP). Сначала вместо интервалов используется выпуклая комбинация левых и правых значений интервальных коэффициентов, и, следовательно, задача сводится к задаче линейного детерминированного программирования. Затем мы преобразуем задачу CCMOSILP в детерминированную задачу, используя известную функцию распределения случайных величин. Основная идея фазы вычислений алгоритма состоит в том, чтобы решить проблему, используя последовательность все более ограниченных целочисленных линейно-дробных программ, которые постепенно улучшают значение линейных критериев и исключают нежелательные моменты из дальнейшего рассмотрения. Для демонстрации предложенного алгоритма решается численный пример.

Ключевые слова: дробное программирование, многокритериальное стохастическое целое, программирование с вероятностными ограничениями, интервальные коэффициенты, эффективное множество.

Цитирование: Юнси-Аббаси Л., Мулай М. Оптимизация дробно-линейной функции с интервальными коэффициентами по целочисленному эффективному набору при вероятностных ограничениях. Вычислительные технологии. 2023; 28(2):72–88. DOI:10.25743/ICT.2023.28.2.007.